DOI 10.1007/s11223-018-9983-8
Strength of Materials, Vol. 50, No. 3, May, 2018

FINITE ELEMENT ANALYSIS OF THERMOELASTIC FIBER-REINFORCED ANISOTROPIC HOLLOW CYLINDER WITH DUAL-PHASE-LAG MODEL

A. D. Hobiny,[a,1] **I. A. Abbas,**[a,b,2,3] **and F. Berto**[c,4]

UDC 539.4

In the present paper, we have constructed the equations for generalized thermoelasticity of a fiber-reinforced anisotropic hollow cylinder. The formulation is applied in the context of dual-phase-lag model. An application of hollow cylinder is investigated for the outer surface is traction free and thermally isolated, while the inner surface is traction free and subjected to thermal shock. The problem is solved numerically using a finite element method. The results of displacement, temperature and radial and hoop stress are obtained and then presented graphically. Finally, the comparisons are made between the results predicted by the coupled theory, Lord and Shulman theory and dual-phase-lag model in presence and absence of reinforcement.

Keywords: dual-phase-lag model, fiber-reinforced, Lord and Shulman theory, finite element method.

Introduction. Materials such as resins reinforced by strong aligned fibers exhibit highly anisotropic elastic behavior in the sense that their elastic moduli for extension in the fiber direction are frequently of the order of 50 or more times greater than their elastic moduli in transverse extension or in shear. Due to their low weight and high strength, the fiber-reinforced composites are used in a variety of structures. The mechanical behavior of many fiber-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fiber direction. The theory of strongly anisotropic materials has been widely discussed in the literature, Belfield et al. [1] investigated the stress in plates reinforced by fibers lying in concentric circles. Hashin and Rosen [2] studied the elastic moduli for fiber-reinforced materials.

The first of such modeling is the extended thermoelasticity theory of Lord and Shulman [3], who introduced the concept of thermal relaxation time into the classical Fourier law of heat conduction. Subsequently, modifying the stress versus strain relationship as well as the entropy relationship with relaxation time, Green and Lindsay [4] proposed the temperature rate-dependent thermoelasticity (GL) theory. The theory was extended for anisotropic body by Dhaliwal and Sherief [5]. Tzou [6, 7] proposed the dual-phase-lag (DPL) model, which describes the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. The DPL model proposed by Tzou [8] is such a modification of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with two different time translations: a phase-lag of the heat flux t_q and a phase-lag of temperature gradient t_θ. Abouelregal [9] studied a problem of a semi-infinite medium subjected to exponential heating using a dual-phase-lag thermoelastic model. Verma [10] studied the shear waves in self-reinforced bodies. Singh [11] discussed the wave propagation in thermally conducting linear fiber-reinforced composite materials with one relaxation time. Othman and Abbas [12]

---

[a]Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia ([1]ahobany@kau.edu.sa). [b]Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt ([2]ibrabbas7@science.sohag.edu.eg; [3]ibrabbas7@gmail.com). [c]Department of Mechanical and Industrial Engineering, Norwegian University of Science and Technology (NTNU), Trondheim, Norway ([4]filippo.berto@ntnu.no). Translated from Problemy Prochnosti, No. 3, pp. 37 – 48, May – June, 2018. Original article submitted March 1, 2017.

studied the effect of rotation on plane waves at the free surface of a fiber-reinforced thermoelastic halfspace. Abbas [13] investigated the effect of magnetic field on thermoelastic interaction in a fiber-reinforced anisotropic hollow cylinder. Chattopadhyay and Choudhury [14] investigated the propagation, reflection and transmission of magneto-elastic shear waves in a self-reinforced media. Chattopadhyay and Choudhury [15] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Tian et al. [16], Abbas et. al [17–23], applied the finite element method in different generalized thermoelastic problems.

In the present paper, we have considered a problem of dual-phase-lag model on generalized thermoelasticity of a fiber-reinforced anisotropic hollow cylinder. The problem has been solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement, radial stress and hoop stress are represented graphically. The results indicate that the different between the coupled theory (CT), Lord and Shulman (LS) theory, and DPL model are very pronounced.

Basic Equations and Formulation of the Problem. For a fiber-reinforced linearly thermoelastic anisotropic medium, the constitutive equations preferred to whose direction is that of a unit vector **a** [11]:

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki})$$

$$+ \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\delta_{ij}, \qquad i,\ j,\ k,\ m = 1,\ 2,\ 3, \tag{1}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i,\ j = 1,\ 2,\ 3. \tag{2}$$

The equation of heat conduction under DPL model [9]

$$\left(1 + t_\theta \frac{\partial}{\partial t}\right)(K_{ij}T_{,ij}) = \left(1 + t_q \frac{\partial}{\partial t}\right)(\rho c_e \dot{T} + T_0 \beta_{ij}\dot{u}_{i,j}), \qquad i,\ j = 1,\ 2,\ 3. \tag{3}$$

The equation of motion

$$\tau_{ij,j} + F_i = \rho \ddot{u}_i, \qquad i,\ j = 1,\ 2,\ 3. \tag{4}$$

Three cases arise:

(i) classical dynamical coupled theory

$$t_\theta = t_q = 0;$$

(ii) LS theory

$$t_\theta = t_0 > 0, \qquad t_q = 0;$$

(iii) DPL model

$$0 < t_\theta < t_q,$$

where u_i are the displacement vector components, ρ is the mass density, e_{ij} is the strain tensor, T is the temperature change of a material particle, τ_{ij} is the stress tensor, β_{ij} is the thermal elastic coupling tensor, c_e is the specific heat at constant strain, T_0 is the reference uniform temperature of the body, t_q is a phase-lag of heat flux, t_θ is a phase-lag of temperature gradient, K_{ij} is the thermal conductivity, α, β, $(\mu_L - \mu_T)$ are reinforced anisotropic elastic parameters, and λ and μ_T are elastic parameters and the component of the vector **a** are $(a_1,\ a_2,\ a_3)$, where $a_1^2 + a_2^2 + a_3^2 = 1$.

Let us consider a fiber-reinforced hollow cylinder with an external radius b and internal radius a. By using the cylindrical system of coordinates $(r,\ \theta,\ z)$ with the z-axis lying along the axis of the cylinder. Due to symmetry, the displacement vector has the components

$$u_r = u(r,\ t), \qquad u_\theta(r,\ t) = 0, \qquad u_z(r,\ t) = 0. \tag{5}$$